

- If $f(x) = x+7$ and $g(x) = x-7$, $x \in \mathbb{R}$ find $f \circ g$ (7)
- Let $*$ be a binary operation defined by $a*b = 2a+b-3$ find $3*4$.
- If $f: \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = \frac{2x-7}{4}$ is an invertible function. Find f^{-1}
- Let $*$ be a binary operation on \mathbb{N} given by $a*b = \text{L.C.M.}(a,b)$ for all $a, b \in \mathbb{N}$. Find $5*7$.
- Evaluate $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$.
- Solve for x , $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}3x$
- Write the principal value of $\cos^{-1}(\cos 7\pi/6)$
- Using principal value evaluate the following: $\sin^{-1}\left[\sin \frac{3\pi}{5}\right]$
- Find the principal value of $\tan^{-1}(-1)$.
- Evaluate $\cos^{-1}\left(\cos \frac{2\pi}{3}\right) + \sin^{-1}\left(\sin \frac{2\pi}{3}\right)$
- Find x if $\begin{vmatrix} 3x & 4 \\ 6 & 1 \end{vmatrix} = 0$
- Find the cofactor of a_{13} in $\begin{vmatrix} 3 & 0 & 5 \\ 4 & -2 & 4 \\ 1 & 3 & 2 \end{vmatrix}$.
- If $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find x and y .
- If $A = \begin{vmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{vmatrix}$ Show that $A^2 = A$
- Let $P = \begin{vmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{vmatrix}$, then show that $PP^T = I_2$.
- Using cofactors of elements of second row Evaluate $\begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$
- Using the properties of det. Solve for x .

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$
- Show that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$ Where a, b and c are in A.P.
- Evaluate $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$
- If $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, write the minor of the element a_{23} .
- Express the matrix $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

22. Find the value of K for which the function $f(x) = \begin{cases} Kx + 5 & \text{if } x \leq 2 \\ (x - 1) & \text{if } x > 2 \end{cases}$ is continuous at $x=2$.

23. Find the value of K if $f(x)$ is continuous at $x=0$

$$f(x) = \begin{cases} \frac{1 - \cos^2 x}{2x^2} & x \neq 0 \\ K & x = 0 \end{cases}$$

24. If $y = \left\{ x + \sqrt{x^2 + a^2} \right\}^n$ prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2 + a^2}}$

25. If $X = 3\sin t - \sin 3t$

$Y = 3 \cos t - \cot 3t$ find dy/dx at $t = \pi/3$

26. Differentiate $\frac{\sqrt[3]{x}}{e}$

27. For the curve $y = 3x^2 + 4x$. Find the slope of the tangent to the curve at the point whose x coordinate is -2

28. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at $x=1$

29. $\int_0^{\pi/4} \log(1 + \tan x) dx$

30. $\int \frac{x}{(x^2 + 1)(x - 1)} dx$

31. $\int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{(1 - x^2)^{3/2}} dx$

32. If $\int (e^{ax} + bx) dx = \frac{e^{4x}}{4} + \frac{3x^2}{2}$, find the value of a and b.

33. $\int_1^3 x dx$

34. If $\int_0^1 (3x^2 + 2x + K) dx = 0$, find K.

35. Find the area enclosed between the curve $y = 4x^2$ the y-axis and lines $y=1, y=4$

36. Find the area enclosed between the curve $y = \sqrt{1-x}$ and the coordinate axes.

37. Write the order and degree of Diff. equation

$$\left(\frac{d^2 y}{dx^2} \right)^2 + \left(\frac{dy}{dx} \right)^3 + 2y = 0$$

38. Solve $(x-1) \frac{dy}{dx} = 2x^3 y$

39. Solve $(x+2) \frac{dy}{dx} = 4x^2 y$

40. Solve $(1+x^2) \frac{dy}{dx} + y = \tan^{-1} x$

41. Find a unit vector in the direction of $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

42. For what value of λ are the vectors are for to each $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular to each other.

43. Find the value of the following:-

$$\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$$

44. Write the direction cosines of a line parallel to z-axis.

45. Find the distance of the plane $3x - 4y + 12z = 3$ from the origin.

46. Maximise $z = 10x + 6y$ subject to the constraints $3x + y \leq 12, 2x + 5y \leq 34, x \geq 0, y \geq 0$.

47. Two cards are drawn from a pack of cards. Find the probability of getting one jack and one queen.

48. Events E and F are independent. Find P(F) if P(E)=0.35, find P(EUF)=0.6

49. If $P(A) = \frac{5}{26}, P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$ find P(AUB).

50. $\int_0^{\pi/2} \frac{\sin x dx}{1 + \cos x}$.

Question Bank (Maths)
(Average)

- Let T be the set of all Triangles in a plane with R as a relation in T given by $R = \{(T_1, T_2) : T_1 \cong T_2\}$. Show that R is an equivalence relation.
- Let * be a binary operation on set Q of rational numbers given as $a * b = (2a - b)^2$, $a, b \in Q$. Find $3 * 5$ and $5 * 3$. Is $3 * 5 = 5 * 3$?
- Let $f: \mathbb{N} \rightarrow \mathbb{N}$ be defined by

$$f(n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases} \quad \text{for all } n \in \mathbb{N} \quad \text{Find whether the function } f \text{ is bijective}$$

4. Prove that $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right) = \pi$

5. Solve for x $\tan^{-1}(2x) \tan^{-1}(3x) = \frac{\pi}{4}$.

6. Solve $\tan^{-1}\left(\frac{1+x}{1-x}\right) = \frac{\pi}{4} + \tan^{-1} x$, $0 < x < 1$

7. Prove that $\sin^{-1}\left(\frac{8}{17}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \cot^{-1}\left(\frac{36}{85}\right)$

8. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ Then show that $A^3 - 23A - 40I = 0$

9. Show that matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = 0$ and hence find A^{-1} .

10. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, Prove that $A - A^T$ is a skew symmetric matrix.

11. Prove that $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy + yz + zx)$.

12. Prove that $\begin{vmatrix} 1 & bc & bc(b+c) \\ 1 & ca & ca(c+a) \\ 1 & ab & ab(a+b) \end{vmatrix} = 0$

13. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \dots + \infty}}}$ then show that $(2y-1) \frac{dy}{dx} = \frac{1}{x}$.

14. Find $\frac{dy}{dx}$ if $\sqrt{x} + \sqrt{y} = 5$.

15. If $x = at^2$, $y = 2at$ find $\frac{d^2y}{dx^2}$.

16. Verify applicability of Rolle's Theorem.

$$f(x) = \frac{x^3}{3} - \frac{5x^2}{3} + 2x, [0, 3]$$

17. Verify L.m.v. theorem if $f(x) = (3x^2 - 2)$, $[2, 3]$

18. Find the approximate value of $f'(3.02)$ if $f(x) = 3x^2 + 5x + 3$

19. Use diff. find the approximate value of $\sqrt{49.7}$

20. Find the intervals in which the function $f(x)=2x^3 - 6x^2 - 48x + 17$ if (a) increasing (b) decreasing

21. Find the coordinate of the point at which the tangent to the curve $y=2x^2 -x + 1$ is parallel to the line $y = 3x+9$.

22. The radius of a circle is increasing at 0.7 m/s. What is the rate of increase of its circumference when $r=4.9$ cm.

23. Evaluate $\int \frac{(6x+7)dx}{\sqrt{(x-5)(x-4)}}$

24. Evaluate $\int \frac{dx}{5+3\cos x}$

25. Evaluate $\int \sin 5x \sin 3x dx$

26. Evaluate $\int_0^{\pi/2} \frac{\sin x dx}{\sin x + \cos x}$

27. Evaluate $\int_4^8 \sqrt{x^4 - 15x^2} dx$

28. Evaluate $\int_0^{\pi/4} \log(1 + \tan \theta) d\theta$

29. Evaluate as limit of a sum (i) $\int_1^3 x dx$ (ii) $\int_0^2 (x^2 + 1) dx$

30. Find the area intercepted between the line $3x - 2y + 12=0$ and parabola $y=\frac{3x^2}{4}$.

31. Find the area of the region bounded by $x^2=16y$, $y=1$, $y=4$ and the y -axis in the 1st quadrant.

32. Find of the region enclosed by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$

33. Find the area bounded by the curve $y=\sin x$ and the x -axis from $x=0$, $x=2\pi$.

34. Find the diff. equation corresponding to the equation $\frac{x}{a} + \frac{y}{b} = 1$.

35. Show that $y=A \cos + B \sin x$ is a solution of the diff. $\frac{d^2 y}{dx^2} + y = 0$

36. Solve $\frac{dy}{dx} + y = \cos x$.

37. Solve $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$.

38. $x \frac{dy}{dx} = y + x e^{-y/x}$.

39. If $|\vec{a}| = 5$, $|\vec{b}| = 13$ and $|\vec{a} \times \vec{b}| = 25$ find $\vec{a} \cdot \vec{b}$.

40. Find the angle between the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ where $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + \hat{j} - 2\hat{k}$.

41. Find the foot of perpendicular from the point (0,2,3) on the line

$$\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$$

42. Find the angle between the line $\frac{x+1}{2} = \frac{3y+5}{9} = \frac{z-3}{6}$ and $\frac{x-1}{2} = \frac{3y-5}{3} = \frac{z-4}{4}$

43. find the image of the point (3,5,3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

44. The vector equation of two lines are

$$\vec{r} = \hat{i} + 2\hat{j} - 3\hat{k} + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$$

$$\vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu(2\hat{i} + 3\hat{j} + \hat{k})$$
 find the shortest distance between the above line.

45. A company manufactures two types of toy A and toy B. Type A required 5 minutes for cutting and 10 min each for assembling. Type B requires 8 min each cutting and 8 min each for assembling. There are 3 hours available for cutting and 4 hours available for assembling in a day. The profit is Rs.50 each on type A and Rs.60 each on type B. How many toys of each type should the company manufacture in a day to maximize the profit?

46. The probability of hitting a target by A is 1/5, If he fired 5 times, find the probability that he will hit atleast two times.

47. Find mean μ and variance for the σ^2 following distribution.

X	0	1	2	3
P(x)	1/8	3/8	3/8	1/8

48. Evaluate $\int \frac{e^x dx}{\sqrt{5 - 4e^x - e^{2x}}}$

49. Evaluate $\int \frac{\cos^5 x dx}{\sin x}$

50. If $(\cos x)^y = (\sin y)^x$ find dy/dx

Question Bank (Maths)
(Above Average)

1. Let $f: \mathbb{N} \rightarrow \mathbb{N}$ be defined by

$$F(n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases} \quad \text{for all } n \in \mathbb{N} \quad \text{Find whether the function } f \text{ is bijective.}$$

2. Consider the binary operation \wedge on the set $\{1,2,3,4,5\}$ defined by $a \wedge b = \min\{a,b\}$. Write the operation table of the operation \wedge .

3. If $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$ and $\beta = \sin^{-1}\left(\frac{12}{13}\right)$, find $\cos(\alpha + \beta)$.

4. Prove that $\tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right) = \frac{\pi}{4} - \frac{x}{2}, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

5. Using the properties of determinants prove that
$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$$

6. A school wants to award its students for the value of Honesty, regularity and Hard work with a total cash award of Rs.6000. Three times the award money for Hard work added to that for Honesty amounts to Rs.17000. The award money given for honesty and hard work together is double the one given for Regularity. Represent the above situation algebraically and find award money for each value using matrix method. Apart from these value namely, honesty, regularity and hard work, suggest one more value which the school must include for awards.

7. If $f(x)$, defined by the following is continuous at $x=0$, find the values of a and b .

$$f(x) = \begin{cases} \frac{\sin(a+1)x + 2 \sin x}{x} & ; \text{if } x < 0 \\ 2 & ; \text{if } x = 0 \\ \frac{\sqrt{1+bx} - 1}{x} & ; \text{if } x > 0 \end{cases}$$

8. Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left[\frac{5x + 12\sqrt{1-x^2}}{13}\right]$

9. If $x = 3\sin t - \sin 3t$ $y = 3\cos t - \cos 3t$, find $\frac{d^2y}{dx^2}$ at

$$t = \frac{\pi}{3}$$

10. If $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$, find $f'(x)$ also find $f'\left(\frac{\pi}{2}\right)$.

11. Find the equation of tangent and normal to the following curves at the indicated points. $x^{2/3} + y^{2/3} = 2$ at $(1,1)$

12. The combined resistors is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ where R_1 and R_2 are respective resistances of two resistors

with $R_1 + R_2 = K$, Where $R_1, R_2 > 0$ and K is constant. Show that maximum resistance R is obtained by choosing resistors for which $R_1 = R_2$

13. Use differentials to find approximate value of $(0.009)^{1/3}$.

14. Show that the semi-vertical angle of the cone of maximum volume and given slant height is $\tan^{-1}\sqrt{2}$?

15. Prove that $x = y^2$ and $xy = K$ cut at right angles if $8K^2 = 1$.

16. Differentiate $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r.t $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

17. Differentiate $x^{\sin^{-1}}$ w.r.t. $\tan^{-1}x$

18. Show that the function $f(x) = x^3 - 3x^2 + 4x$, $x \in \mathbb{R}$ is strictly increasing on \mathbb{R} .

19. The perimeter of a triangle is 16 cm. If one side is 6cm, find the other two sides so that the area of triangle is maximum.

20. Find the interval in which the function f given by $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$, is strictly increasing or strictly decreasing.

21. Evaluate $\int \frac{\sin \theta \cos \theta d\theta}{\cos^2 \theta - \cos \theta - 2}$

22. Evaluate $\int \frac{e^x dx}{e^x(e^x - 1)}$

23. $\int \frac{\cos^9 x}{\sin x} dx$

24. $\int \frac{x^3 + x + 1}{x^2 - 1} dx$

25. Evaluate $\int_1^3 (x^2 - x + 5) dx$ by as limit as sum

26. Evaluate $\int_1^4 (x + e^{2x}) dx$ by as limit as sum

27. Using the properties of definite integrals evaluate $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}}$

28. Find the area of the region bounded by the graph of $y = \sin x$ and $y = \cos x$ between $x = 0$ and $x = \pi/2$

29. Find the area of the region $\{(x, y): x^2 + y^2 \leq 1 \leq x + y\}$.

30. Find the area of the region enclosed between the circles $x^2 + y^2 = 1$ and $(x - 1)^2 + y^2 = 1$.

31. Show that $y = be^x + ce^{2x}$ is a solution of the differential equation $\frac{d^2 y}{dx^2} - \frac{3dy}{dx} + 2y = 0$

32. Solve $\frac{dy}{dx} = xy + x + y = 1$.

33. Solve $x \frac{dy}{dx} - y + x \sin\left(\frac{y}{x}\right) = 0$

34. Solve $e^{\frac{dy}{dx}} = x + 1; y(0) = 3$.

35. Find the projection of vector $\vec{a} + \vec{b}$ on the vector \vec{a} where $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{k}$.

36. If \vec{a} and \vec{b} are unit vectors and θ is the angle between them then show that $\sin\left(\frac{\theta}{2}\right) = \frac{1}{2} |\vec{a} - \vec{b}|$.

37. Prove that the lines $\vec{r} = \hat{i} + \hat{j} + \hat{k}$, and $\vec{r} = 4\hat{i} - \hat{k} + \mu(2\hat{i} + 3\hat{k})$, intersect.

38. Find the equation of the plane which contains the line of intersection of the lines of intersection of the planes

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$$

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0 \text{ and which perpendicular to the plane}$$

$$\vec{r} = (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$$

39. Ram wants to invest at most Rs.12,000 in saving certificate and Bonds. According to rules, he has to invest at least Rs.2000 in certificates and atleast Rs.4000 in Bonds. If the rates of interest in certificates and Bond are 8% and 10% per annum respectively. How much money should he invest to earn maximum yearly income. Also find his maximum yearly income.

40. In a bolt factory machines A, B and C manufacture 25%, 35% and 40% respectively of the total production of their output 5%, 4% and 2% are respectively defective bolts. A blot is drawn from the production and is found to be defective. What is the probability that it is manufactured by machine B?

41. A coin is tossed 5 times. Find the probability of getting atleast 3 heads.

42. A coin is tossed six times find the probability of getting 4 or more heads.

43. Suppose that 90% of people are right handed. What is the probability that at most 6 of a random sample of 10 people are right handed?

44. Evaluated $\int \frac{x^4 dx}{(x-1)(x^2+1)}$

45. Evaluated $\int \frac{x^2 dx}{(x^4 + x^2 + 1)}$

46. If $x = a(\cos t + t \sin t)$
 $y = a(\sin t + t \cos t)$
 $0 < t < \pi/2$

Find $\frac{d^2x}{dt^2}$, $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$

47. Prove that $\sec^2(\tan^{-1}2) + \operatorname{cosec}^2(\cot^{-1}3) = 15$

48. If $\sec^2(\tan^{-1}x) + \operatorname{cosec}^2(\cot^{-1}x) = 27$ find x .

49. Prove that $\tan^{-1}2 + \tan^{-1}3 = 3\pi/4$

50. Using the properties of definite integral

(i) $\int_0^{\pi/2} \log \sin x \, dx$

(ii) $\int_0^{\pi/4} \frac{(\sin x + \cos x) \, dx}{9 + 16 \sin 2x}$

(iii) $\int_0^{\pi/2} 2 \log \sin x - \log \sin 2x \, dx$

51. $\int \sqrt{\tan x} \, dx$

52. $\int \sqrt{\cot x} \, dx$

53. $\int \sqrt{\tan x} + \sqrt{\cot x} \, dx$

54. If $\int \frac{dx}{\cos^4 x + \sin^4 x}$

55. $\int_{-3}^3 |x+1| \, dx$